How Price Instability Complicates the Analysis of Price Supports

Bruce Gardner

Agricultural economists often invoke instability, risk, and uncertainty as central concepts in understanding farm policy. Yet in analyzing policy alternatives we typically use comparative static methods that shift supply and demand curves and find new equilibria without reference to uncertainty in the economy or economic behavior other than profit-maximizing (risk neutral). This is true even in the most complex models being used to analyze 1985 farm bill alternatives, such as Johnson, et al. (1985). The aim of this paper is to understand this combination of simultaneous emphasis and neglect of risk considerations in policy analysis, and to assess costs of ignoring risk. The discussion consists of two parts, the first on normative considerations and the second on positive economics and risk.

The Goal of Stabilization

At one level, the practical normative economics of stabilization consists of statements of politicians and agricultural experts about the goals of agricultural policy. Prominent among the goals, indeed chief among them in some lists, is the idea of ensuring consumers of the availability of food and of defending farmers against developments causing disastrously low returns. The goals can be summarized as a desire to provide stability of food prices and farm incomes.

At a second level, even more practical normative economics is implicit in goals as revealed by the consequences of enacted policies. The goals revealed in this way are better described as price and income support than as stabilization. The distinction is that stabilization moderates the tails of the frequency distributions of prices and returns, reducing the frequency of both extremes, while support is concerned only with raising low prices and returns. The revealed goal of U.S. agricultural policy is farm income support. Congress in the past 50 years has never to my knowledge enacted a piece of legislation intended to bring down unusually high farm returns. The closest approach in recent years was perhaps July 1975, when bakers were testifying that bread prices would reach $1 per loaf if the Soviet Union were allowed to buy more of the U.S. wheat crop, at a time when food prices were rising at double-digit annual rates. Congress not only did not act but when President Ford did attempt downward stabilization through a hold on grain sales to the Soviets, the reaction of Congress was to pass legislation intended to prevent the Executive Branch from ever doing such a thing again. Presidents, in 1975, and also with the meat price freeze and soybean embargo of 1973, have attempted downward price stabilization on rare occasions, but it has been politically costly. President Carter halted grain sales to the Soviets in January 1980, not to bring prices down but to punish the Soviets in the matter of Afganistan. He made every attempt to ensure that farmers suffered no losses in consequence. But he still paid a substantial price politically.

In short, stabilization as a policy goal is a mirage. We may think we see it, but it is not there. Still, policies do affect price and income instability, even if in an asymmetrical manner. The situation is further complicated in that changes in policy may themselves be a source of instability, as several agricultural economists have emphasized in recent years.

The normative issue from an analytical viewpoint is the incorporation of instability and risk into welfare economics. In what sense can commodity price instability constitute a market failure, and how do we measure the social costs? My assessment of this subject appears elsewhere (1985) and will not be restated here. Instead, let us turn to some issues in positive economics in the presence of commodity market instability. The discussion is at

Professor of Agricultural and Resource Economics, University of Maryland.

an elementary level, but quicky leads to complicated problems.

Price Supports Under Instability

For concreteness let us characterize instability as randomness in price received by producers caused by unpredicted variability in demand, e.g., export demand for grains, or in agricultural output. The introduction of a price support program can be analyzed at the following levels: (1) effects on (the frequency distribution of) prices facing a farmer, (2) effects on a farmer's profits or returns, (3) effects on a farmer's utility, and (4) effects on market equilibrium. In the existing literature effects (1) are well known in general although practical procedures for calculating effects of price supports are not well developed. Effects (2) have been well developed. Effects (3) have been explored much less, but may nonetheless have been overplayed. Effects (4) have hardly been investigated but are the most important effects.

1. The first step in analyzing price supports under uncertainty is to replace the concept of equilibrium or expected price by a frequency distribution of producer prices. Certainty is then a special case in which the frequency distribution degenerates to a spike at the mean price with zero frequency for all other prices. In this special case a support price has no effect if the support price level, P_s, is below the market equilibrium level, P_e. In the uncertainty case, the support price truncates the frequency distribution at P_s, and so has an effect even if $P_s < P_e$.

How can we quantify the effect? If we consider the frequency distribution of price, we want to know, first, the difference between the mean of the underlying distribution, P_e, and the mean of the distribution when the tail with $P < P_s$ is eliminated. This expected gain, E(AP), is

(1) $$E(AP) = \int_{P_s}^{\infty} Pr(P)\,P\,dP - \int_{0}^{\infty} Pr(P)\,P\,dP.$$

We rule out negative prices and assume that Pr(P), the probability that price is at level P, for all $P \geq P_s$ is the same whether the lower tail is truncated or not. This implies that the integrals from P_s to infinity cancel out the expected price change is:

(2) $$E(AP) = P_s Pr(P_s) - \int_0^{P_s} Pr(P)\,P\,dP$$

where $Pr(Ps) = \int_0^{P_s} Pr(P)\,dp$, i.e., the cumulative probability of prices below P_s in the absence of price supports. The additive terms in equation (2) can be collected to yield:

(3) $$E(AP) = \int_0^{P_s} Pr(P)\,(P_s - P)\,dP.$$

This is the value of an option to sell at P_s, i.e., a put option with strike price of P_s. This value can be readily approximated if we know the frequency distribution of prices below P_s.

Even more simply, if we are willing to assume that the commodity price is normally distributed, we have

(4) $$E(AP) = \int_0^{P} TCoV5w)^{-1} e^{-w \cdot -W(\cdot * r12} (P_s - P)\,dP$$

The integral is taken from 0 since negative prices do not occur; but since a normal distribution of P can generate negative prices, the lognormal distribution is preferable. For practical purposes, however, the assumption of normality may not make much difference and it makes the algebra a little simpler. (The reader who wishes to see results for lognormal prices is referred to Gardner 1977.)

Equation (4) can be simplified for calculating purposes by the trick of expanding (P_s — P) to (P_s - P - P H- P), dividing the integral into two parts and converting to a standard normal form by setting Z = (P — P)/o\ This implies that dP — o-dZ. The manipulations yield:

(5) $$E(AP) = (P_s - P) \int_0^{Z_s} \frac{L}{\sqrt{2\pi}} e^{-z^2/2}\,dZ - \int^{2} \frac{i}{\sqrt{2\pi}} e^{-z^2/2} o\text{-}dZ$$

$$= (P_s - P)\,F(Z_s) + (o/\sqrt{2\pi})e^{-z^2/2}$$

where F is the cumulative normal density function and $\mathbf{Z_s}$ is the normalized support price, (P_9 - P)/or.

For example, suppose that for soybeans P = \$6.00 per bushel, the standard deviation of price is \$.80, and the support price is \$5.60. This means that Z_s = (5.60 - 6.00)7.80 = - .5. Since the cumulative normal probability to — .5 is .309, we have:

$$E(P) = -.4\,(.309) + (.8/2.507)e^{-.25/2} = \$.16$$

Thus, we can estimate the expected price gain from a support price below the mean price. Note that as *a*- -» 0, E(AP) -> P_s - P for P_s > P and zero for P_s ^ P.

2. The effects on a farmer's expected profits are to a first approximation straightforward. Profits rise by the amount of the expected revenue gain. However, we need to consider a producer's supply response to an increased expected price. The increased costs associated with output expansion must be subtracted from the expected revenue increase. The net increase is the increase in producers' surplus.

Following Oi (1961), there have been many studies of the effects on profits of variability in price when the mean price is unchanged. The findings generally are that price variability makes producers better off, assuming that producers can respond to random price changes after they occur. For example, if export demand increases, and hence output price rises, producers can increase production to take advantage of the enhanced profit opportunities. In such models the stabilization element of a support price makes producers worse off, so that a static analysis overstates their gains.

If variability takes the form of uncertainty, in which producers must choose a desired production level before the random price is known, the effect of variability price on profits is ambiguous. If a producer's output is uncorrelated with price, then a change in variability leaves expected revenue unchanged, and since costs are the same for all outcomes, expected profits are unchanged. If price and output are correlated, then expected profits may increase or decrease depending on the functional form of the demand function, the form of disturbances and the correlation coefficient between the farm's output and market price (see Just, Hueth, and Schmitz, Chapter ID-

3. The effects on a farmer's utility are the same as the effects on profits if utility is a linear function of profits, i.e., if the marginal utility of profits is constant. This condition is equivalent to the farmer's being risk neutral. If the farmer is risk averse, i.e., the marginal utility of profits declines as profits increase, the stabilization element of the price support program can make producers better off even if their expected profits were to decline. Helms (1985) illustrates this point with examples for consumers. The usual result for producer support prices would be that risk aversion implies producer gains even greater than the expected profit gains would indicate. Moreover, if consumers are highly risk averse and the commodity supported is large enough in consumers' budget shares, it is possible that consumers as well as producers can be made better off by a price support program. This is, however, unlikely to be an important point in practice, as suggested by the results of Helms' simulation. The most extreme risk aversion he considers is an Arrow-Pratt relative risk aversion coefficient of 6, which implies that consumers would give up about 12 percent of their (mean) income in exchange for stabilizing an income stream that, unstabilized, would have a coefficient of variation of .2 (i.e., expected deviation from mean income is 20 percent of mean income). Helms finds that even in this case consumers are worse off when price is stabilized at *5* percent above its mean value.

4. The preceding are at best partial results because they do not consider change in market equilibrium caused by price supports. Analyzing the consequences of price supports in this context is a problem in comparative stochastic statics—stochastic because randomness is incorporated in the model, but static in that we look at stationary mean values rather than adjustment paths overtime. Some complications in such analysis are apparent in even the simplest models. Consider a 2-state model with linear supply and demand. Let the (inverse) demand function be:

(6) $$P = a_0 - aj\ Q$$

Let the supply function be

(7) $$Q = b_0 + bjP^* + v$$

where P* is expected price and v takes on constant values rt v, each with probability .5. For a first approximation to market equilibrium, let us use the usual "rational expectations" specification. The approach defines equilibrium as equality of producers' anticipated price P* with the (statistical) expectation of price P (see Turnovsky). The equilibrium is found by taking expectations of (6) and (7) and solving:

$$P = a_0 - a_t [b_0 + bj\ P]$$

fQ\ W f> _ ªO ~~ 3-ibo
r — -—•------ —
$1 + a_t bi$

which is the intersection of demand and mean supply. Now suppose we introduce a price support level P_s, at which the government buys all that the market will not take at P_s. This means that the demand function becomes perfectly elastic at P_s. Now P will be P_s when v is positive and at the market clearing price when v is negative. The average of prices at the two outcomes is:

P_5 = 1/2 {a_0 - a! [b_0 + bi P_s - v]} + 1/2 P_s

Solving for P,

(9) P = 1/2 [a_0 - a_l(b_0 - v) + PJ,

PL < P_s < PH

where P_L is P when v > 0 and P_H when v < 0. P_s < P_L has no effect. For P_s > P_H, P_s = P_s. Equation (9) enables us to calculate the mean price associated with any price support level. Consider the particular price support level where P_s equals the mean price with no intervention, so that in a static framework the program would have no effect. Substituting for this level of P_s from (8) into (9),

HQI P = vai / ('2 + a^O

In order to see what this amounts to, figure 1 shows a special case in which a_0 — 13, a! = .1, b_0 — 70, bi = 10, and v = 10. This implies a mean supply-demand intersection at P — 3 and Q = 100. The price support at P_s = 3 implies P_s = 3.33 and Q_s = 103.3, plotted as point R in figure 1. From equation (10) we see how the expected price differs from the support level, and how this difference depends on v. In particular, defining AP = P_s — P_s, we have

(11) $\frac{d(AP)}{dv}$ = $\frac{ai}{2 + aibi}$

which is positive for normal-sloped demand and supply. Thus point R rises along the mean supply curve as v increases.

This first approximation does not generate full competitive equilibrium, however. Consider the example. We have the following price-quantity pairs under the support program: ifv- -10, thenP, Q is (3.67, 93.3), and if v - +10, (3.00, 113.3). This implies that re venue is 342.4 if v = - 10 and 339.9 if v = 10, for a mean of 341.15. If this is to be represented as competitive equilibrium at point R, then P • Q on the mean supply curve should have the same value. But 3.33 x 103.3 = 344.0. Therefore R is not the equilibrium. In fact, P = 3 and Q = 100 is not the equilibrium with no program! With no program the P, Q pairs are (3.50, 95) if v = - 10 and (2.50, 105) if

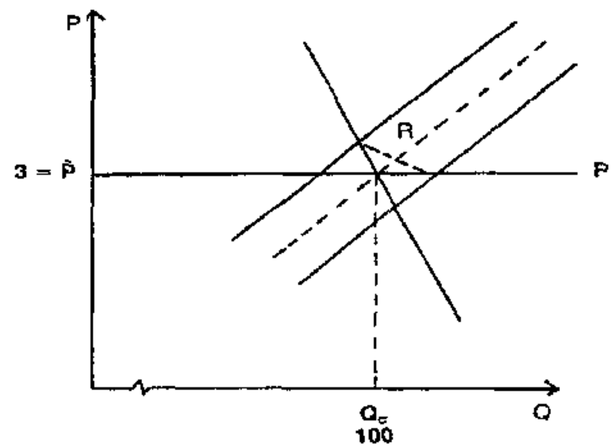

Figure 1. Price Support at Mean Price

v = +10. Mean revenue is 1/2 • 332.5 + 1/2 262.5 = 297.5, which is less than 3 • 100. An industry-wide expectation of loss is not consistent with competitive equilibrium.

The problem is the specification of the rational expectations equilibrium as P = P*. For (risk-neutral) models of Turnovsky and other work following Muth (1961), we need a zero-profit condition for equilibrium. In expected value this is

(12) E(PQ) = E(P*Q),

or expected revenue equals expected economic cost. Expected economic cost is an opportunity cost concept, the receipts necessary to induce producers to provide the quantity of expected output that consumers will purchase. The implication of this equilibrium condition for the linear two-state model of equations (6) and (7) is that we cannot simply set P — P*. Instead, we have, where Q is expected output,

(13) E(R) = E(PQ)
- I/2(a_0 - ai(Q + v))(Q + v) +
I/2(a_0 - ai(Q - v))(Q - v)

and

(14) E(P*Q) = -£-° Q + 4- Q^2-
bi D!

Equation (14) is obtained by solving (7) for P*, expressing it in terms of intended output (omitting v), and multiplying by Q. Intended output is assumed equal to (statistically) expected output—a condition for equilibrium. This specification implies that producers can-is substituted into (6) and solved for P. The relevant distinction has been labelled as being between price variability and uncertainty. Variability refers to a situation such as pre-planting wet weather which affects planting-season price expectations and hence current-year output. Uncertainty refers to a situation in which the farmer makes production decisions and only afterwards does the random shock occur, e.g., an August drought in corn. Equations (6)~ (7) model variability, and (13)-(14) uncertainty.

Equating (13)-(14), obtains the quadratic

(15) $(-1 + _{EI})\ Q^2\ 4\text{-}\ g\_^{\circ} + a_0)\ Q + a_{IV}^2 = 0.$

For the parameter values above, (15) is

$.2Q\text{-}^2 - 20\ Q + 10 = 0,\ Q = 99.5.$

The resulting equilibrium is shown in figure 2 most straightforwardly as the intersection of total cost and expected revenue at point E. From the upper panel, this is *not* the quantity at which the intersection of intended supply and demand. The appropriate depiction of competitive equilibrium in P, Q space is the intersection of the expected average revenue (EAR) function with the intended supply function. The EAR function specifies, for every intended quantity Q*, the expected revenue per bushel. This would be a point on the demand curve only if the total revenue function were linear which would only be if the inverse demand function were of the form, P — a 4- $BQ\text{-}^1$).

Because EAR is less than the demand price at each quantity for a linear demand curve (quadratic total revenue function), competitive equilibrium is characterized by less output than at the supply-demand equilibrium. This implies that mean price will be above the price at supply-demand intersection. In figure 2, the price under uncertainty is \$3.05 per bushel and EAR is \$2.95 at the competitive equilibrium Q of 99.5, while stationary supply equals demand at P = 3.00 and Q = 100.

For risk-averse consumers and/or producers the analysis of full equilibrium is more complex. We need to know how revenue translates to income, and then how income translates to utility. Following on the preceding example, suppose that half of the costs of intended output are purchased inputs, the other half are returns to producer-owned in-

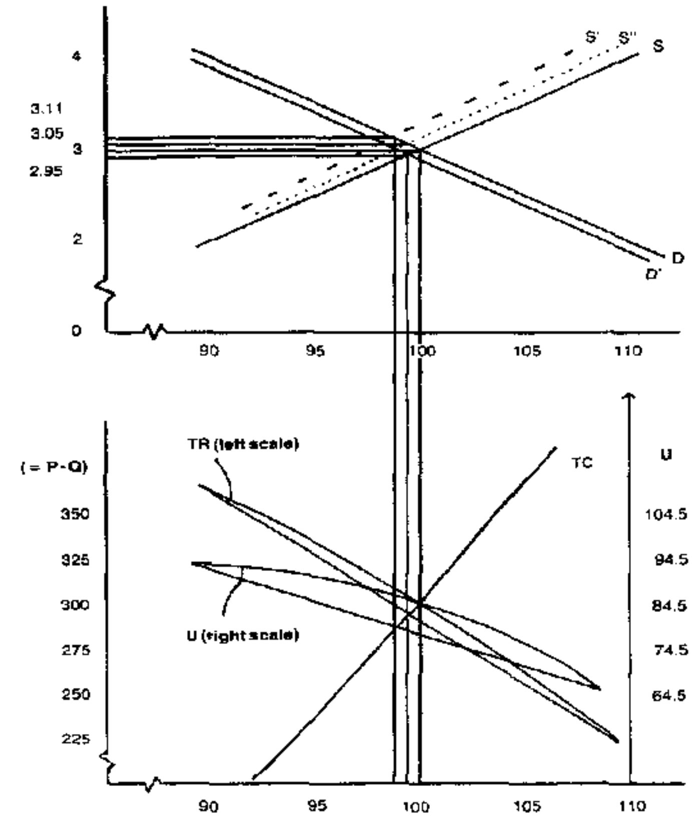

Figure 2. Equilibrium with EAR Less Than Demand Price

puts. Suppose further that these returns constitute half of producers' income, the other half being nonstochastic income from some other source. This means that in the 2-state example with equilibrium as shown in figure 2, we have: when v = -10, Q = 89.5, P = 4.05, PQ = 362.48; and when v = 10, Q = 109.5, P = 2.05, PQ = 224.48; which implies that mean net returns are 293.48/2 = 146.74. Under the assumptions, there is an equal amount of nonstochastic income, so total income, y, is 293.48. Since all the variation in revenue is residual income to the farmers, when v = —10, y = 362.48 and when v = 10, y — 224.48. The coefficient of variation of y is .236.

Continuing the example with utility in the two states, suppose all the farmers are identical and have equal shares of market income, and all have the same utility function, of the constant-risk-aversion form,

(16) $U(Y) = C + (1 - RK'y^1\text{-}^*,$

where C is a constant and R is the relative risk-aversion coefficient U"(Y)/U' (Y) Y. Let R = 1.5, so that U(Y) = *C - 2/VY.* This degree of risk aversion implies that producers would exchange the example's income stream for a stable income stream that was 4 percent smaller.

To find competitive equilibrium under risk aversion we equate the expected utility gain from the uncertain income from producing the crop to the (certain) utility loss from producing it. The utility loss is simply the utility of the cost as given by equation (14) which in our example is:

$$U(P^*Q) = C - 2\backslash\hat{}\ \underset{l\_bi}{Q} + \pounds s\ \underset{bi}{Q^2}\ \underset{J}{1\sim^*}.$$

Similarly, the expected utility gain is obtained from (13), and we find the market equilibrium by equating (13) and (14), as modified, solving for Q.

$$(17) \quad C - 2\underset{l\_bi}{\&-^\circ}\ Q + \underset{Dl}{J-}\ Q^*\underset{J}{]}\sim^* = 4\underset{2}{-}<^{C} - {}^{2/}K^{a\circ}$$

$$4- {}_{ai}(Q + e))(Q + e)]-^*\}$$

$$+ -I\{C - 2/[(a_0 + a/Q - e))$$

$$x\ (Q- e)]-^*\}$$

Note that C cancels out, so that R is the only parameter of the utility function that affects the solution for Q. Using the parameters of the figure 2 example, equation (17) reduces to

$$2(.1Q^2- 7Q)\sim-^5 + (-.1Q^2 + 1\_1Q + 120)--^5$$
$$+ (-.1Q^2 + \overline{1}5Q - 140)--^5= 0$$

which solves for Q = 98.9. This implies a mean price of $3.11. As shown in figure 2, risk aversion approximately doubles the output decline and price rise that occurred under risk neutrality, as compared to the supply-demand intersection (certainty case).

In the way risk analysis is typically applied, the entire difference between certainty outcomes and results under uncertainty is attributed to risk aversion. For example, one could draw a supply curve through the joint where P = 3.11 and Q = 98.9, as shown by the curve, S', in figure 2. The shift from the certainty case is then interpreted as an indicator of risk aversion. But this sort of "shift" occurs even without risk aversion; in the example only about half the output reduction occurs because of risk aversion. It might be less open to misinterpretation to represent equilibrium uncertainty by using the EAR function to show the (risk-neutral) revenue effect and a shift in S to represent the risk-aversion effect, as in the dotted curve S". This intersection of D' and S" then shows full equilibrium under uncertainty .

Conclusions

What is the empirical significance of the four types of complications? Consider the corn program in the 1985 farm bill. Several in Congress have proposed freezing the 1986 target price at the 1985 level of $3.03 per bushel. Others, including the Reagan Administration want to reduce the target price by making it a declining function of past prices, e.g., 100 percent of the past 5-year average in 1986, 95 percent in 1987, etc. The farm prices of corn for the past five years are: $3.11, $2.50, $2.68, $3.25, and $2.65 per bushel (the last figure being USDA's estimate for 1984/85). The 5-year average is $2.83. The practical question for policy analysis is: what difference will it make in 1986 if the target price for corn is $3.03 or $2.84?

Supposing that $2.84 is the appropriate mean price, we can calculate from equation (4) the increase in expected price caused by increasing the target price to $3.03 if we know the parameter *a-* (assuming normality). The sample standard deviation of price for the past five years is $.16 per bushel. Using these values in equation (4), the expected producer price gain is $.20, as compared to the $.19 that a deterministic approach would assume. The accuracy of estimates of both P and cris questionable , but the difference between the stochastic and the crude deterministic estimates is too small to cause excitement. (But of course for support prices below mean price, the stochastic estimate of price effect can be substantially greater than the zero effect that a deterministic model gives.)

Bringing in expected profits or producers' surplus gains requires knowledge of the supply function, and cost components of farmer-owned and purchased inputs. Our information here is weak, but this difficulty applies as much to policy analysis in a deterministic as in a stochastic framework.

Bringing in risk aversion for farmers and consumers is important if: (a) they are significantly risk averse, *and* (b) the commodity in question accounts for a significant fraction of farm income or consumers' budgets, *and* (c) market participants do not manage risk by non-program means, e.g., hedging, diversification, insurance. We really do not know enough about any of these factors to assert that policy analysis that ignores risk aversion and just adds up monetary gains and losses is misguided.

Bringing in market equilibrium occupied most of the discussion in this paper. It becomes a factor in applied policy analysis only when we have already dealt with the preceding micro-level complications, and with additional serious problems, not discussed here, with aggregating over diverse individuals. Given the difficulties of undertaking these preliminary steps, the full stochastic competitive equilibrium methods discussed are not in the cards for analyzing policy alternatives in the 1985 farm bill. Nonetheless, it is good to be aware of what is being ignored when we ignore uncertainty doing comparative statistics of the usual kind. The main practical lesson is that even "low" price supports can have substantial effects. For policy research, the bottom line is the difficulty of separating out the consequences of risk aversion from the (risk neutral) effects of variability on expected profits, a subject on which the authors of estimates of farmers' risk aversion coefficients or supply shifts due to risk aversion have been unduly optimistic about their capabilities.

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